under axial pressure contains about 50% membrane stress, he concludes that $\sigma^c = \frac{1}{2}\sigma^c_{(\text{classical})}$ is a lower bound estimate and rules out the influence of boundary conditions. However, I have calculated the membrane contribution for the critical load of the free edge cylindrical shell following Croll and found that according to his theory a lower bound must be $\sigma^c = \frac{1}{4}\sigma^c_{(\text{classical})}$ and not $\sigma^c = \frac{1}{2}\sigma^c_{(\text{classical})}$. It is also interesting to note that imperfection sensitivity in some axially inextensional structures can exist due to nonlinear compatibility conditions and regardless of the amount of membrane strain energy as pointed out in detail in Refs. 4 and 5.

Having said that, we must, of course, admit that the works of the second and third groups have, nevertheless, illuminated many dark corners and clarified for us many ambiguous points.

References

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Comment on "Inviscid Hypersonic Flow around a Semicone Body"

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KIMURA and Tsutahara propose in Ref. 1 a solution for the inviscid hypersonic flow past a flat topped semicone body. Having established by an unsteady analogy that the cross-flow velocity components obey the Laplace equation they proceed to solve this under boundary conditions which include an assumption of vanishing velocities at infinite radius. They then combine the known density ratio across shock waves at infinite Mach number with the continuity equation to obtain the shock shape from the calculated velocities. Since this interposes a discontinuity between the body and the boundary condition at infinity, one is inclined to suspect the relevance of that condition. In fact, that boundary condition can only be satisfied because a genuinely relevant condition has been omitted. Consider a conical shockwave $r/x=r_s(\theta)$, normal to which a vector in (x,r,θ) coordinates may be written as $\boldsymbol{n}=(r_s,-1,r_s^{-1}\mathrm{d}r_s/\mathrm{d}\theta)$. The equation for continuity of mass may be written $\rho_1(\boldsymbol{q}_1\cdot\boldsymbol{n})=\rho_2(\boldsymbol{q}_2\cdot\boldsymbol{n})$ or

$$v_\theta\frac{\mathrm{d}r_s}{\mathrm{d}\theta}+r_s^2\left[v_x-\frac{\gamma-1}{\gamma+1}U\right]-v_r r_s=0 \tag{1}$$

The equation for continuity of momentum parallel to the shock is

$$\boldsymbol{q}_1\times\boldsymbol{n}=\boldsymbol{q}_2\times\boldsymbol{n}$$

which yields two independent equations

$$U-v_x-v_r r_s=0 \tag{2}$$

$$v_r\frac{\mathrm{d}r_s}{\mathrm{d}\theta}+v_\theta r_s=0 \tag{3}$$

Equation (2) may be interpreted as $v_x=U+0(r^2/x^2)$, which is intuitive anyway, and combined with Eq. (1) to yield

$$v_\theta\frac{\mathrm{d}r_s}{\mathrm{d}\theta}+\left[1-\frac{\gamma-1}{\gamma+1}\right]r_s^2-v_r r_s=0 \tag{4}$$

which is Eq. (10) of Ref. 1, and is employed there to find the shock shape. Equation (3), however, is not considered at all, although it has quite equal status with Eq. (4) as an identity which must be satisfied on the shock. The condition for (3) and (4) to be compatible is

$$v_r^2+v_\theta^2=2v_r r_s/(\gamma+1) \tag{5}$$

It is this boundary condition, to be applied on the initially unknown shock-wave, which should replace the spurious boundary condition at infinity. It should be noted that Eq. (3), and hence Eq. (5), is automatically satisfied in axisymmetric flow ($v_\theta=0$), or two-dimensional flow ($v_\theta=v_r\tan\theta$), and it would seem safe to neglect it in situations close to one of these. However, inspection of the results of Ref. 1 indicated a large region, roughly $\pi/4<|\theta|<\pi/2$, where neither assumption is satisfied. Thus, although the flow in $|\theta|<\pi/4$ may be self-consistent, it cannot be regarded as caused by removing the cone top, since the regions in which cause and effect operate are separated by a region in which the flow model breaks down.

Reference

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Reply by Authors to P. L. Roe

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IT is natural that the condition for Eq. (3) in Roe's comment must be considered. However such momentum relations are sometimes neglected in constant density solutions.[1] For a complete solution, the relations between the flows of the two sides, before and behind the shock wave, such as the conservation equation for energy and the relations of entropy change, must be satisfied, as well as Eq. (3). Since the strength of the shock wave cannot be the same everywhere, they are contradictory to our assumption that the internal energy and entropy are uniform within the shock layer. Therefore we do not think that the previous assumption leads a satisfactory solution for the whole flowfield.

Mr. Roe says that it is Eq. (5) in his Comment, to be applied on the initially unknown shock wave, which should replace our spurious boundary condition at infinity. However, it is difficult to think that this will improve our analysis, because the contradictions in the region where θ is large are essentially

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from the assumption of constant density and homentropy. So the solution, which satisfies Eq. (5) without changing the assumption, has physically no sense. On the other hand, it is not unnatural to think that the flowfield in the shock layer has the same pattern as the flow with the boundary condition that $v \to 0$ at infinity, since in two-dimensional flow the semicircular piston expands in a fluid at rest.

With regard to the last statement of Roe's Comment, we believe that even when the analysis is improved by taking the density and entropy changes into consideration, if the density and entropy do not change very much (as would be true in practice), the flow in the self-consistent region will also not be changed very much.

We would like to express our appreciation to Roe for his Comment. We wish to conclude by saying that although our analysis obviously has some defects, it is not a complete one, but it is the first for higher approximations.

Reference

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Re-Examination of the Role of Radiation in Hybrid Regression Rate Theory

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SOME studies on hybrid combustion theory have been reported recently by Rastogi et al.[1,2] The principal conclusion of these two papers is that radiant flux is very important in the operating regime of hybrid rockets. In these studies the lower limit of the oxidizer fluxes at which experiments have been conducted is about 1.5 to 1.8 gm/cm^2-sec. Wooldrige and Muzzy[3] show that for most cases of interest, the radiant flux is important only at oxidizer fluxes of the order 0.2 to 0.4 gm/cm^2-sec. Hence one should not expect radiant flux to be important in the experimental regime of the authors.[1,2] After a careful examination of the experimental results reported by Rastogi et al.,[1,2] it appears that their conclusions are incorrect. This Comment is intended to point out the sources of the errors and possible corrective procedures.

The authors[1] consider the regression of a tubular grain through the port of which oxidizer flows. The balance of the heat flux at the surface of the fuel is written as (notations same as in Ref. 1)

$$\rho_f \dot{r} \Delta H = \dot{Q} - \dot{Q}_{re} \qquad \text{[Eqs. (14) of Ref. 1]}$$

where

$$\dot{Q}_{re} = \sigma\, \epsilon_w \epsilon_r T_r^4 \qquad \text{[Eq. (16) of Ref. 1]}$$

and

$$\dot{Q}_c = \text{convective flux}$$

It is stated that $\dot{Q}_r$, the radiant flux to the wall (it is not clear if the wall refers to fuel surface, but is presumed so), is neglected. The expression of $\dot{Q}_{re}$ is obtained from the work of Wooldrige and Muzzy[3] in which it is clearly and correctly identified as the radiant flux emitted by the flame and absorbed by the surface. It has to be added to the convective flux and *not* subtracted as is done in Eq. (14) of Ref. 1; however it has to be suitably reduced to account for the blocking effect. In their analysis they assume that radiant flux is zero at time $t=0$ and convective flux is present. It is not clear why one of these fluxes is zero at $t=0$ and as to why time should enter into the picture in this way in a steady-state problem. The heat balance at the wall should be considered under steady conditions, and when so done, both the terms will be present all the time.

Consequent upon their formulation the authors[1] obtain the relation

$$\log[\rho_f \Delta H/\sigma\epsilon_w p^\beta T_r^4(\dot{r}_t - \dot{r}_c) + 1] = \alpha\, pz \text{[Eq. (23) of Ref. 1]}$$

where z is identified as the diameter of the duct at any time t. It can be observed that the left-hand side (LHS) of this equation is zero at $t=0$ (since $\dot{r}_t = \dot{r}_0$) whereas the right hand side (RHS) has a nonzero value. The equation is therefore inconsistent and hence incorrect. The fact that the authors obtained a nonzero value of $(\dot{r}_t - \dot{r}_c)$ at $t=0$ (Fig. 12 of Ref. 1) raises doubts about their calculation procedure.

The experimental data on regression rate are used to obtain the LHS of Eq. (23) of Ref. 1 for various values of z. The linearity of the plot of LHS vs z of Eq. (23) is taken as a support for the theory. Unfortunately Eq. (23) demands that the plot, LHS vs z, must go through (0,0). As seen from Fig. 11 of Ref. 1 this does not happen.

Finally the term $\dot{Q}_{re}$ with the interpretation that it represents the heat transfer from the flame front to the exhaust gases is irrelevant to the heat balance at the wall. In fact correct heat balance at the wall is[3]

$$\dot{Q}_w = \dot{Q}_c f + \dot{Q}_r \tag{1}$$

where

$$\dot{Q}_r = \sigma\epsilon_w p^\beta \epsilon_r T_r^4 (1 - e^{-\alpha p z})$$

and f is the factor to reduce the convective flux to account for blowing.

The next point is concerned with the representation of the variation of radius with time as[1]

$$r = A + kt^x \qquad \text{[Eq. (25) of Ref. 1]}$$

which gives

$$\dot{r} = kxt^{x-1} \qquad \text{[Eq. (26) of Ref. 1]}$$

It must be noted that both A and k are positive. The range of values of x lies between 0 and 1. x cannot be greater than 1 since this implies that regression rate increases with time, a fact which is contrary to observations (Fig. 2 of Ref. 1). If $x<1$, the regression rate is infinity at $t=0$. However, the plot in Fig. 2 of Ref. 1 shows a finite value for the regression rate at $t=0$. This implies that the curve fit is not appropriate, at least around $t=0$.

One of the aspects of the analysis of regression data is that the relation of curve fit should be given careful attention. The best technique is to presume a relation which has a theoretical basis. For instance in the case studied by Rastogi et al.[1] one can obtain from $\dot{r} = aG^n$ the following approximate relation for r vs t as

$$r^{1+2n} - r_0^{1+2n} = bt \tag{2}$$

Now a suitable curve fit treating n and b as unknowns will lead to values which will not have any singularities. Using the data of Ref. 1 given in Figs. 2 and 12 a replot of r^2 vs t is made and is shown in Fig. 1. It is very nearly a straight line which implies that n is about 0.5, which is true for laminar boundary layers. Equation (2) can be further refined to take

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